Name of College – S. S. College, J. Bad.

Deptt – Mathematics

Topic – Rearrangement of series

Class – B.Sc II

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By – Dr. Shree Nath Sharma
(Associate Prof)

<u>Study Material</u> –

* Conditional and Non-Conditional Convergence :→

A Convergent Series which is unaffected by any rearrangement of its Term is called an Unconditionally Convergent Series. Where as the Series which is thus affected is Called Conditionally Convergent.

Ex:→ The Series

$$1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots$$

is Unconditionally Convergent.

Since it's value is not affected by any rearrangement.

it's value is $2/3$

<u>Now</u> Consider the re-arrangement

$$\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \cdots\right) - \left(\frac{1}{2} + \frac{1}{2^3} + \cdots\right)$$

which is equal to

$$\frac{1}{1-\frac{1}{4}} - \frac{1/2}{1-\frac{1}{4}}$$

$$= \frac{4}{3} - \frac{2}{3} = 2/3$$

The Series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - - - -$$

is conditionally convergent since its value can be affected by re-arrangement.

Re-arrangement of Terms of an absolutely Convergent Series —

If the Terms of an absolutely convergent series are re-arranged the series remain convergent and its sum is unaltered.

or

The sum of absolutely convergent series is independent of the order of Terms.

Proof :→ Let Σu_n be the Given Series and New series after re-arrangement be $\Sigma u_n'$ so that every u and u' and every u' and u.

Since the Series Σu_n is absolutely convergent, there can be found a +ve integer m corresponding to a +ve No ϵ however small such that

$|u_{n+1}| + |u_{n+2}| + \cdots |u_{n+p}| < \varepsilon$

for every $n \geq m$ and for every $p \geq 0$

In particular, we have.

$|u_{m+1}| + |u_{m+2}| + \cdots + |u_{m+p}| < \varepsilon$ —— (1)

for $p \geq 0$

Let $S_n = u_1 + u_2 + \cdots + u_n$

$S'_n = u'_1 + u'_2 + \cdots + u'_n$

Since Σu_n is convergent:

$\lim_{n \to \infty} S_n \to S$

Now there can be choosen a +ve integer $\mu \geq m$ so Large that for all values of $n \geq \mu$, the partial sum S_n and S'_n have the Terms $u_1, u_2 \cdots u_m$ in common.

Let the Terms which are not common in S_n and S'_n be.

$u_{m+\alpha}, u_{m+\beta}, \cdots u_{m+r}$

So that for all values of $n \geq \mu > m$ we have

$|S'_n - S_n| = |u_{m+\alpha} \pm u_{m+\beta} \pm \cdots \pm u_{m+r}|$

$\leq |u_{m+\alpha}| + |u_{m+\beta}| + \cdots + |u_{m+r}|$

$< \varepsilon$ From (1)

i.e $S'_n - S_n \to 0$ as $n \to \infty$

& $\lim_{n \to \infty} S'_n = \lim_{n \to \infty} S_n = S$

Hence the series $\Sigma u'_n$ is convergent and has the Same Sum as the Series Σu_n